1. Introduction

Recent observations of distant Supernovae reveal that the expansion of the universe is in an accelerated phase at the present time, which indicates that our universe may be dominated by a negative pressure component called dark energy. Without a satisfactory model from fundamental physics, many large surveys are proposed to probe the properties of dark energy as accurately as possible, such as the weak-lensing survey LSST.

Light from distant sources will be deflected when traveling through the gravitational potential of the foreground matter distribution, causing their images to be both magnified and distorted. In the weak-lensing regime, the distortions or the so-called cosmic shear fields can be statistically measured from the ellipticities of background galaxies, which potentially provide a powerful probe of the structure of the Universe and hence of the dark energy properties.

On large scales, statistical properties of the cosmic shear fields are fully characterized by the two-point correlation functions as is for a Gaussian random field. While on small scales where non-linear effects have induced non-negligible non-Gaussianities, higher-order statistics are needed to describe the fields. Alternative methods include the statistics of the non-linear structures, such as the abundance of shear-selected galaxy clusters, the fractional area of the high-shear regions etc.

By combining the methods that apply to large scales and small scales, the available information from the LSST weak-lensing survey will be more fully utilized.

We model the redshift distribution of the source galaxies as that inferred from observations by the Subaru telescope with a limiting R band magnitude of 26. Half of the galaxies with $z \in [1.2, 2.5]$ are discarded due to insufficiently accurate photometric redshift from the LSST. The distribution is normalized to 65 arcmin$^{-2}$ (note a more realistic number is 40 arcmin$^{-2}$).

2. Number Counts

Galaxy clusters can be selected from the high-peaks of the shear maps. Their number counts are exponentially sensitive to the amplitude of the matter fluctuation fields and linearly sensitive to the comoving volume element.

We utilize all the clusters in the redshift range of $[0.1, 1.4]$, approximately 280,000 in total. By dividing them into 26 redshift bins, their abundance allows the evolution of the universe to be explored. We found at least 10 bins are needed to avoid degrading the constraints on dark energy parameters.

3. Shear-shear Correlations

Photometric-redshift information available from the LSST enables the background galaxies to be divided into different redshift bins. The constructed shear maps allow the matter distribution to be probed tomographically. We divide the background galaxies with $z \in [0, 3.2]$ into photo-z bins of size $\Delta z_{20}=0.4$. These give us 8 auto-correlations and 28 cross-correlations.

4. Covariance

Since both the number counts of the shear-selected galaxy clusters and the shear-shear correlations are derived from the same weak-lensing survey, we need to consider their covariance when trying to combine these two methods. We assume the galaxy clusters trace the underlying matter distribution through a linear bias factor, and neglect non-correlating sampling errors of both the clusters and the background galaxies. We find the covariance between these two observables

$$\xi_{\ell}(\delta \Delta z, \delta \Delta z') \approx \frac{1}{2} \left( \frac{1}{N} \sum_{i,j} \eta_i(\delta \Delta z, \delta \Delta z') \frac{1}{N} \sum_{i,j} \eta_j(\delta \Delta z, \delta \Delta z') \right)$$

The correlation coefficient is given by

$$\rho_{\ell, \Delta z} = \frac{\int d\Delta z \eta(\Delta z) \xi_{\ell}(\Delta z, \Delta z')}{\sqrt{\int d\Delta z \eta(\Delta z)^2 \int d\Delta z' \eta(\Delta z') \xi_{\ell}(\Delta z, \Delta z')}}$$

Sample variance for the number counts of clusters is dominated by the largest angular modes because of the large survey area of LSST, about half a sky. So its correlation with smaller angular modes (we choose $\ell_{min}=40$) of the shear fields is small.

5. Combination

We use the Fisher Matrix Formalism to forecast the constraints on dark energy properties. We consider 7 parameters, i.e., $\Omega_m h^2, \Omega_c h^2, \Omega_k, \alpha, n_s, w_0$, and $w_a$. We do not write down the details of the Fisher matrix calculation here. For the shear-shear correlations, systematic effects from multiplicative and additive errors in the shear measurements, photometric redshift errors, correlations between the cosmic shear and the intrinsic alignments of the source galaxies and uncertainties in the predicted non-linear matter power spectrum require further explorations from both experimental and theoretical sides. For weak-lensing cluster selection, projection effects along the line-of-sight can be reduced by increasing the detection threshold. Alternatively, one may utilize the properties of shear peaks directly, such as their number counts. Considering the shear-peaks are also non-linear structures, we would expect our findings hold similarly for the combination of the number counts of shear peaks and the shear-shear correlations for the LSST. While the systematic effects will degrade the constraints from each observable alone, their combination may be less affected.

6. Conclusions

We studied the possibility of improving constraints on dark energy properties by combining two observables – the number counts of detected galaxy clusters and large angular scale tomographic shear-shear-correlations – that will both be available in the LSST weak-lensing survey. We calculated the covariance between these two observables, and found it to be small (10^-3 or less) and negligible. We found that the combination results in an improvement of at least a factor of 2 on dark energy parameter uncertainties relative to using either observable alone.