Weak Lensing Cosmology with LSST: Three-Point Shear Correlations

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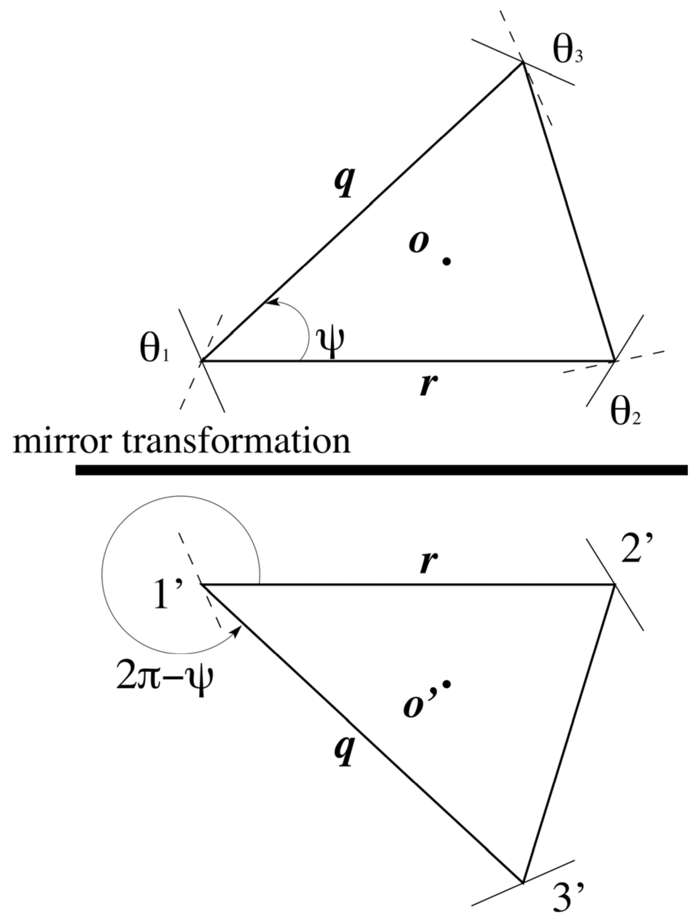
We present an analysis of the three-point correlation function for weak lensing shear data. The shear three-point function is an independent measurement from the two-point function and thus adds to the total signal-to-noise obtainable from weak lensing data. Furthermore, it is shown that the constraints on cosmological parameters are along somewhat different degeneracies than the two-point function, so the combination of the two statistics is significantly more powerful than either one individually. Predictions of the constraining power are given for the proposed Large Synoptic Survey Telescope. We also present the actual marginal detection from the 75 square degree CTIO Lensing Survey and the E/B mode analysis of this dataset.



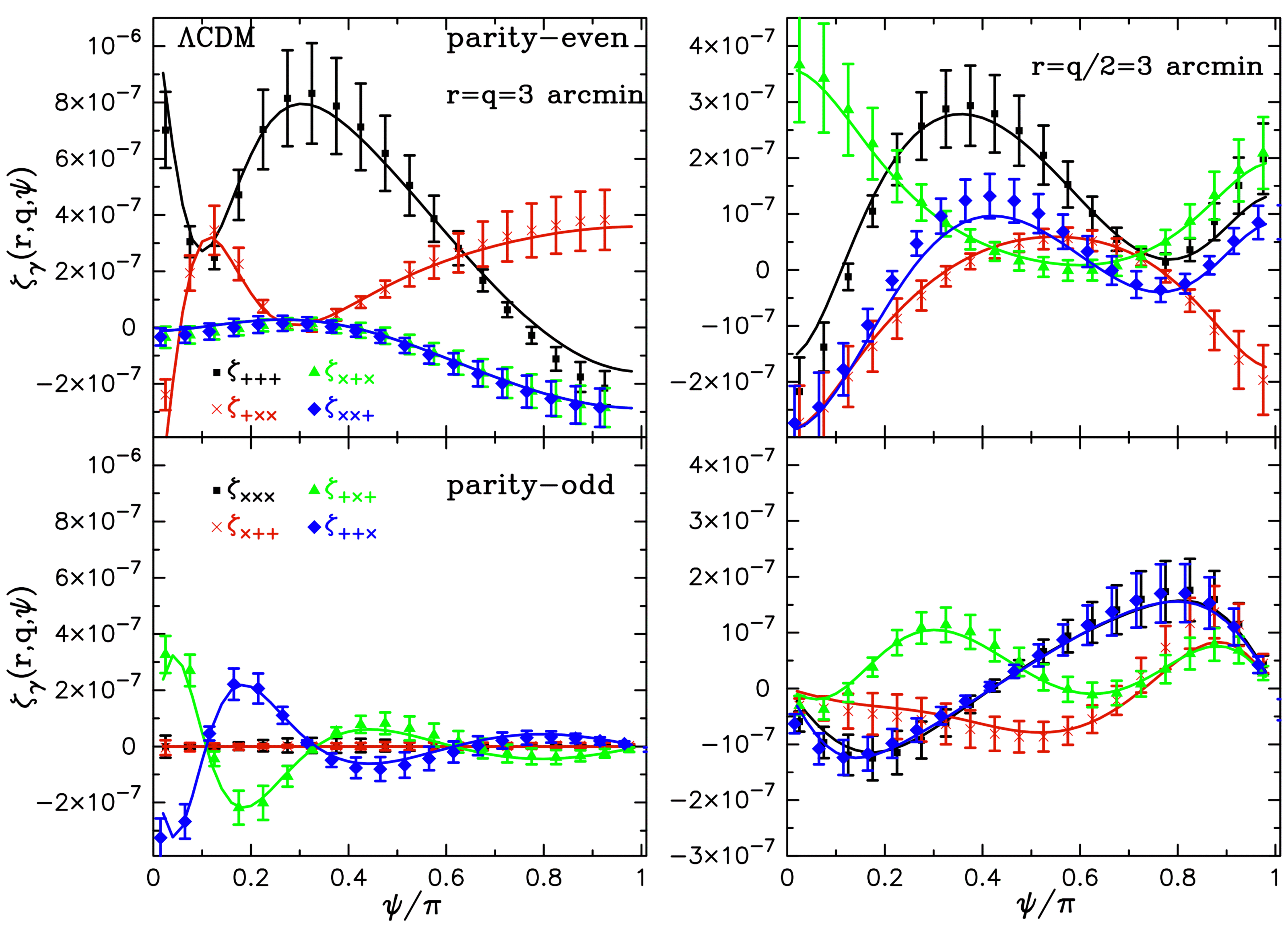
The Three-Point Function

The three-point shear correlation function is distinctly more complicated for weak lensing studies than the two-point function and corresponding derived statistics like the popular aperture mass statistic. However, the extra complications are worthwhile, since three-point function can probe aspects of the shear field, such as non-Gaussianity, which the two-point statistics cannot.

First, geometry of triangles dictates that the three-point function is a function of three parameters: for example (q,r,ψ) in the diagram at left.

Second, the shear at each vertex has 2 components. Thus, the full three-point function has 8 combinations of these, leading to 8 separate correlation functions. These can be divided into parity-odd and parity-even functions corresponding whether they change sign under the transformation $\psi \rightarrow 2\pi-\psi$.

The predicted values of the 8 functions for ΛCDM cosmology are plotted below as a function of ψ for two values of (r,q). + and x refer to the two shear components relative to the center of the triangle.



Discussion

The algorithm we have developed for measuring the three-point correlation function of the shear field is a huge improvement in calculation time over previous algorithms, and will be absolutely critical for a data set as large as that of the LSST. We have also developed a parallel version of the algorithm which has better than 50% of the maximum efficiency when scaled up to 1024 processors. This will also likely be important for its use with the LSST data.

It is clear from the plots at right that the three-point correlation function constrains the cosmological parameters in a slightly orthogonal direction to that of the two-point function. Thus, when the two statistics are combined, the overall contours are much smaller than for either one separately. See the posters by Takada, et al. (108.16), Knox, et al. (108.14), and Haiman et al. (108.15) for more about the weak lens cosmology constraints which will be possible with the LSST, including dark energy parameters.

Also, the contours above are all quite narrow in one direction. Since other experiments will constrain the parameters with different degeneracy directions than these, combining their constraints with the results from the LSST will lead to very tight overall constraints. See the poster by Knox, et al. (108.14) for constraints from combining the LSST data with supernovae.

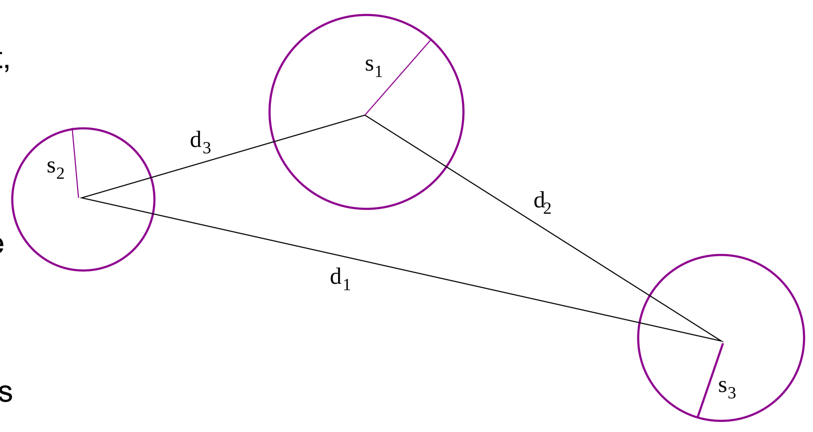
Note that the results presented here assume that the systematic errors in the lensing data can be reduced to the level of the statistical errors. This requires an order of magnitude improvement over these Blanco 4m results. While this is a significant challenge, recent progress in the algorithms for PSF correction and interpolation, as well as the excellent precision of the LSST optics and detectors, give reasons for optimism that the systematics can be controlled to the level necessary. Indeed, recent shear data from new technology telescopes confirm this (see Claver poster 108.10).

Measuring the Three-Point Correlation Function

The direct way to measure the three-point correlation function is to take every possible set of three galaxies, and multiply the various components of the shear, and then put the product into a bin according to the shape of the triangle. This method is impossible in practice for more than a few thousand galaxies, since the computing time scales as N^3, where N is the number of galaxies. We have developed an improved algorithm which asymptotically scales as N ln(N) in the limit of large N. (At $N = 10^5$, the scaling is approximately $N^{1.1}$, already close to the asymptotic limit.)

The algorithm divides the field into a tree of nodes which each contain 1 or more galaxies. When a node contains more than one galaxy, it also keeps pointers to two subnodes which each have half of its galaxies. The top node in the tree has all the galaxies, and the bottom of the tree are leaves with one galaxy each. Each node keeps track of the average position, shear, etc. of its ensemble of galaxies.

The essential trick to speed up the calculation is to use these average values whenever the nodes are small enough compared to the sides of the triangle. In the diagram at right, the s values are the maximum distance from any galaxy to the centroid. The d's are the distances between the centroids of the three nodes being considered. If the size, s, for a node is too large, we split it into its subnodes and try those corresponding trinagles.



The binning of the triangles defines a coarseness to the measurement. Thus, we declare the nodes small enough when the error in the triangle shape from using the averages rather than the component galaxies is at most the size of the bins.

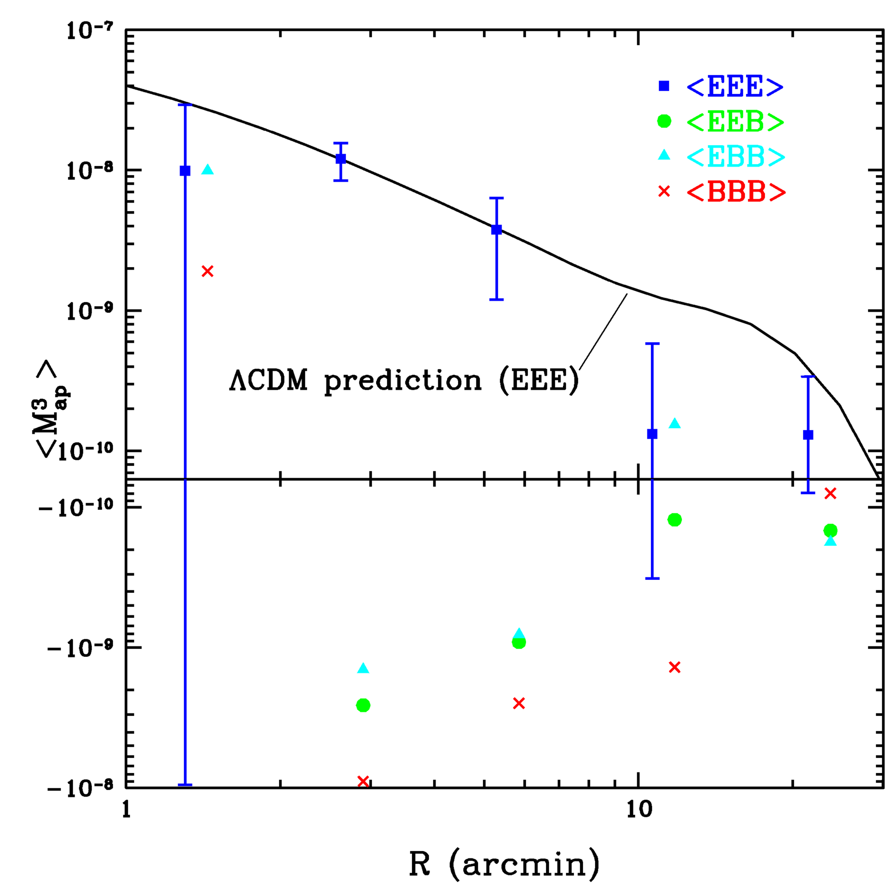
The same strategy leads to fast algorithms for the two-point correlation function as well as higher order correlation functions.

Current Measurements

We have used this algorithm on the CTIO weak lensing survey, which has about 2 million galaxies, and covers about 75 square degrees. The algorithm takes about a day on a desktop machine with reasonable binning of the triangles. (The time is also strongly dependent on the bin sizes, of course, but we expect our choice to remain reasonable for LSST data.) With of order 10^3 times as many galaxies, we expect the calculation time to be quite reasonable for the LSST data with the computers which will be available.

Since the full three-point correlation function is somewhat unwieldy to plot, we make use of the aperture mass cubed statistic, $\langle M_{ap}^3 \rangle$, which combines the measurements into a single function. This function captures most of the lensing information in the three-point data, including only pure E-mode correlation information. There are three similar functions which contain B-mode information (both pure, and mixed). Since lensing produces a pure E-mode shear field, these other functions are useful measures of systematic contaminations in the data.

The results are shown to the right, along with the predicted value for the convergence ΛCDM model. The B-mode and the mixed modes are all consistent with zero, while the E-mode is non-zero at the 95% confidence level, and is consistent with the convergence model.



Predicted Cosmology Constrants with LSST

At right, we show the constraints on cosmological parameters which will be possible with LSST's lensing data. We show the 68% confidence limit contours for the two-point and three-point correlation functions separately (green and gray, respectively), as well as the combined constraints using both measurements (blue).

These constraints take advantage of the LSST's ability to measure photometric redshifts for the lensed galaxies. Not only does this improve the calibration of the source population compared to what is possible with current surveys, but it also allows us to perform separate auto- and cross-correlations of galaxies in different redshift slices. (We use 5 redshift slices here.) The cross-correlations in particular are referred to as tomography. They improve the signal-to-noise in general by a factor of two or so, and they improve the constraints on w_a by a factor of about 10.

We also assume priors on n_s, $\Omega_b h^2$ and h which are expected from the Planck mission. The LSST lensing data is taken over the range of $50 < \ell < 3000$, which roughly corresponds to $5' < \theta < 5°$. The LSST survey data are taken to cover 20,000 square degrees, with a galaxy density of 50 per square arcminute.